Interactive Formal Verification
9*: Presenting Theories*

Tjark Weber
Computer Laboratory
University of Cambridge

Concrete Syntax

- Suggestive textual representation is important:
  - x+y instead of plus x y
  - [1,2,3] instead of Cons 1 (Cons 2 (Cons 3 Nil))
  - 'a × 'b instead of ('a, 'b) prod

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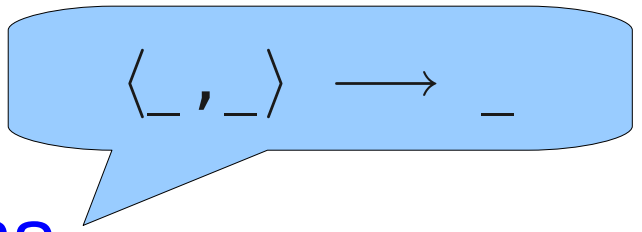

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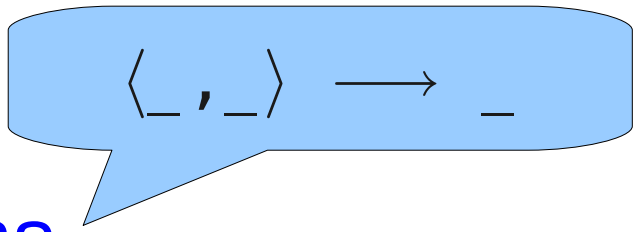

- Isabelle uses mixfix annotations.
- These can be specified wherever constants (or types) are defined: definition, fun, datatype, ...

Infix Annotations

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definition xor :: "bool ⇒ bool ⇒ bool"
  (infixl "[+]" 60)
where "A [+] B ≡ (A ∧ ¬B) ∨ (¬A ∧ B)"
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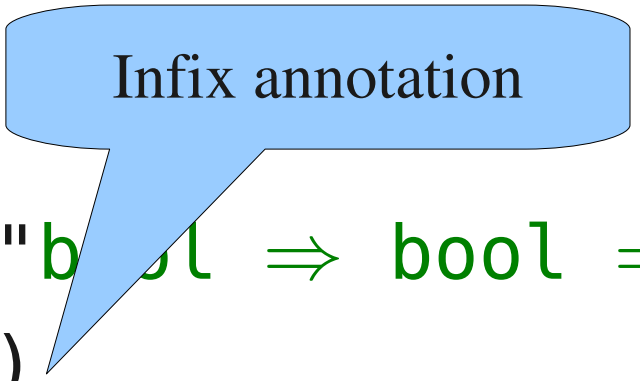


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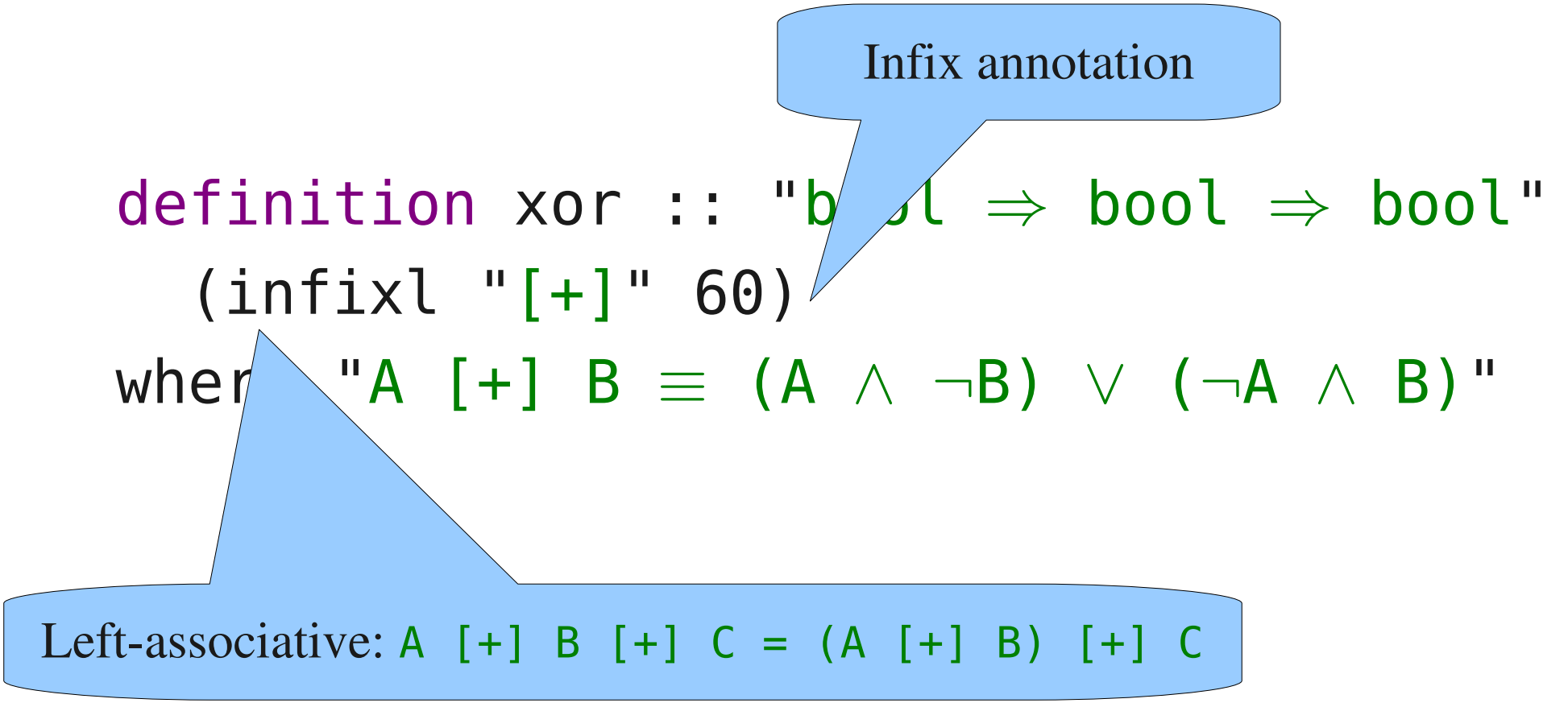

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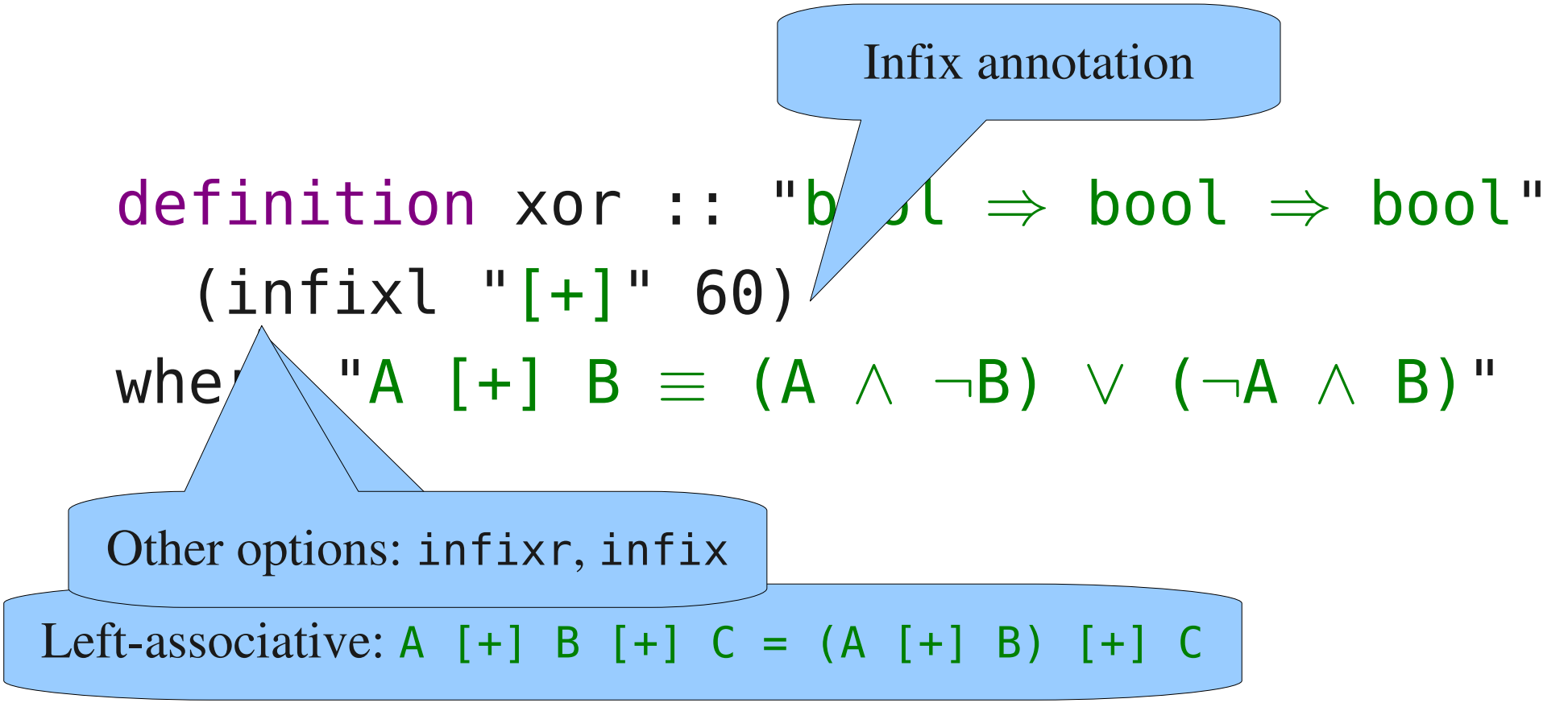

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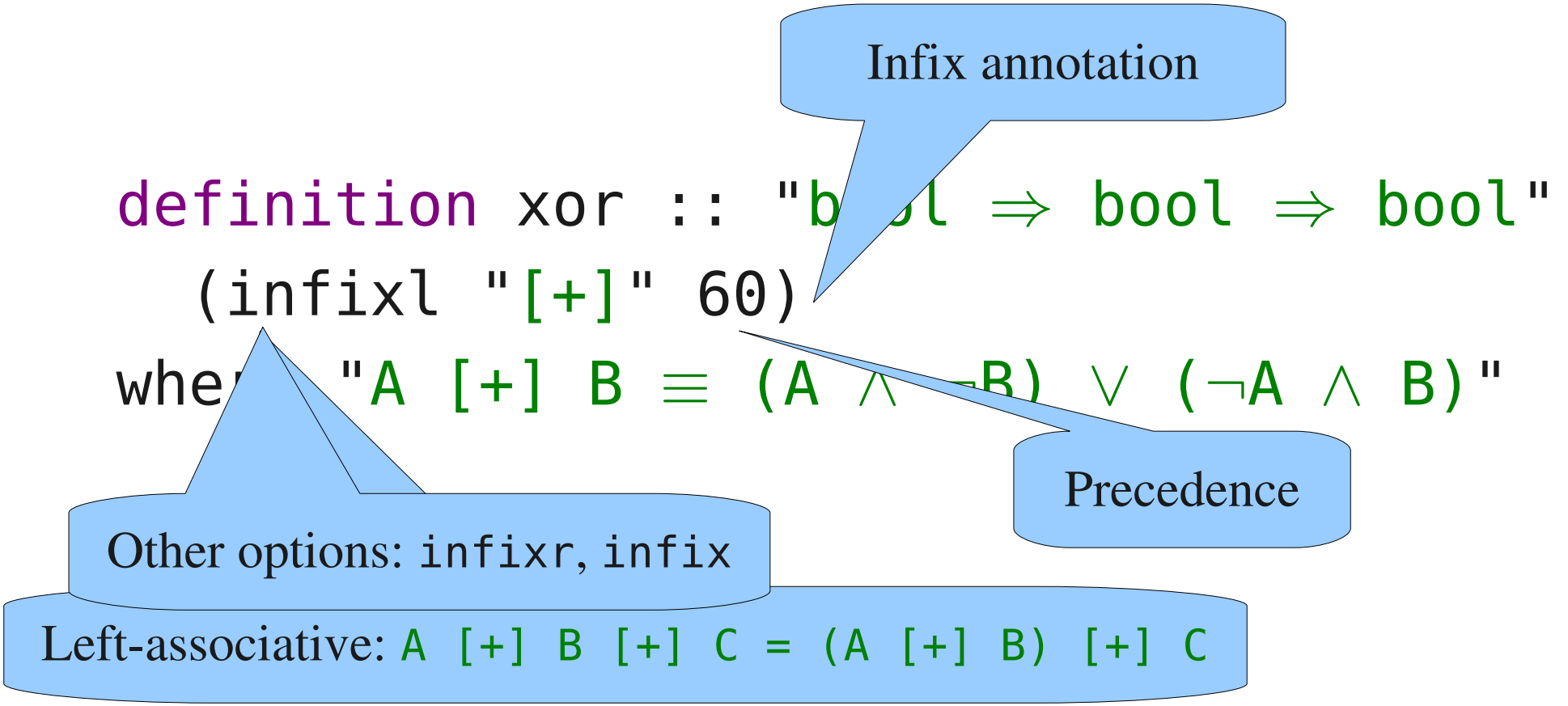

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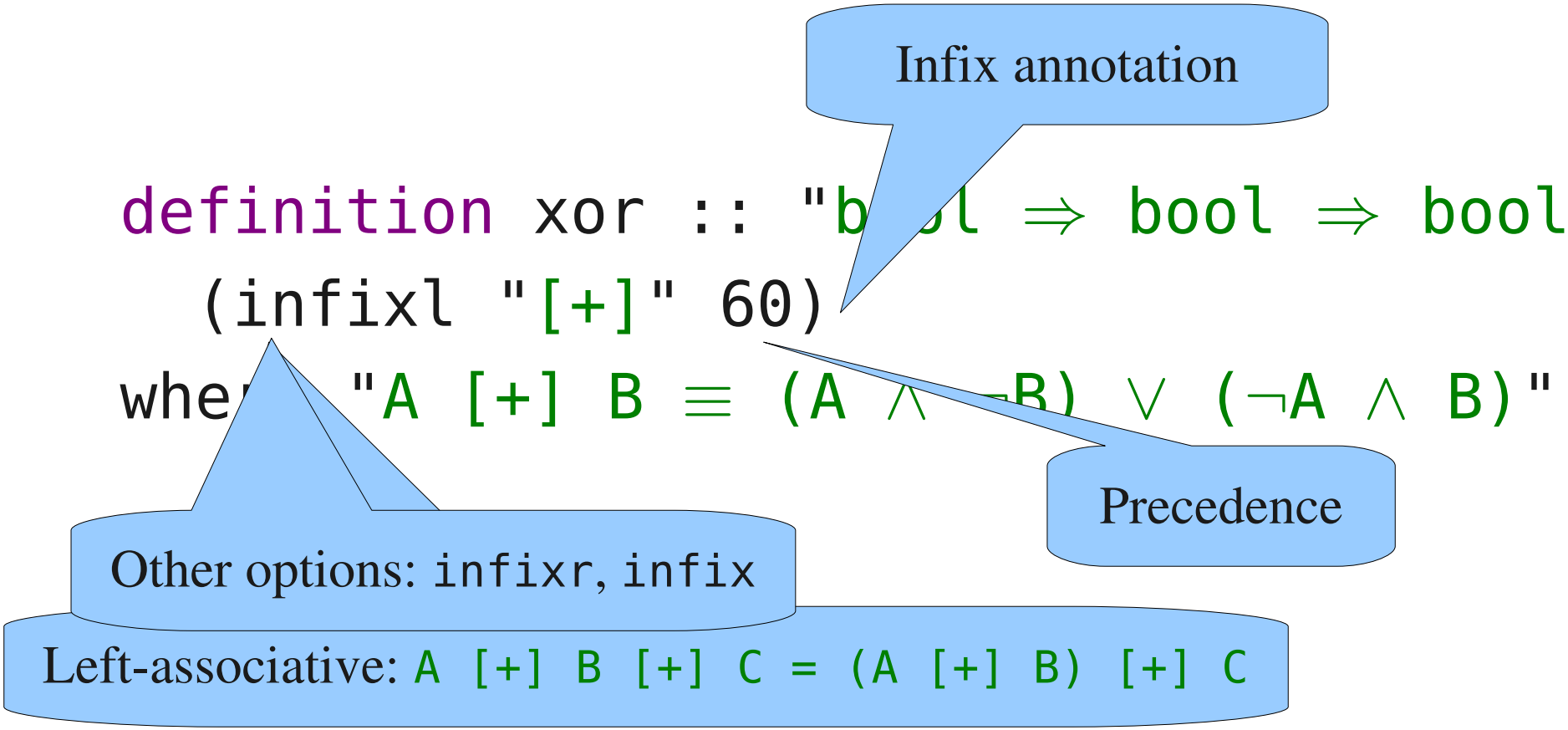

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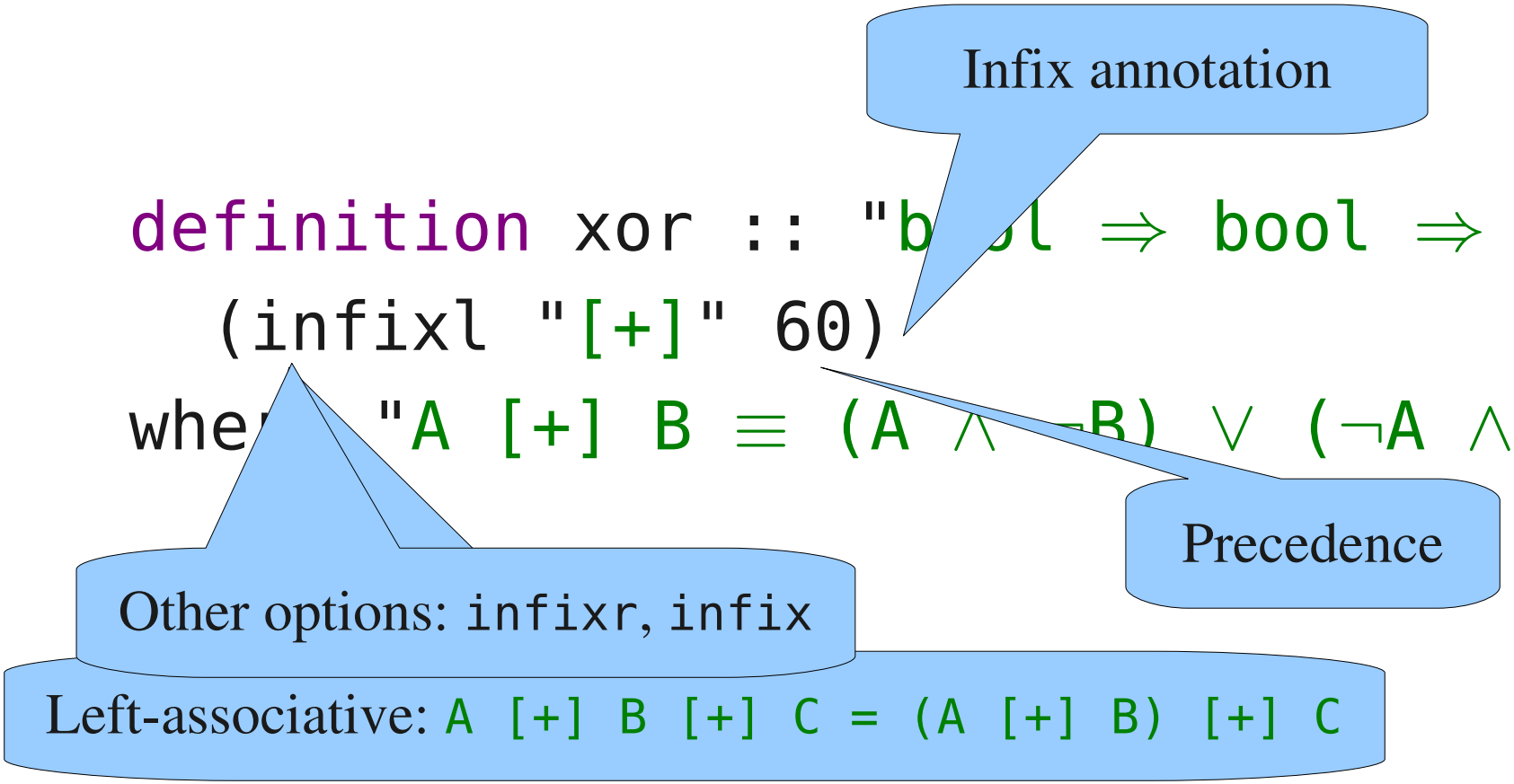

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- Use `op` for partial application: `op [+]`

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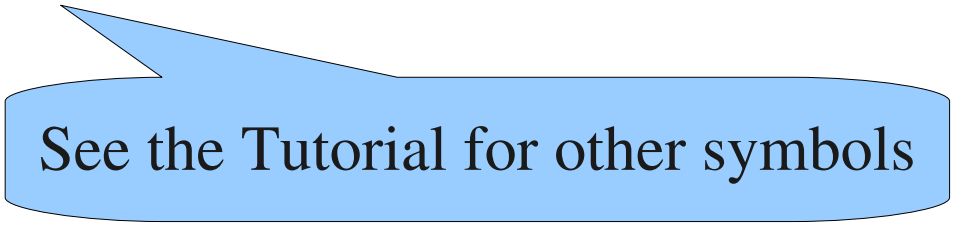

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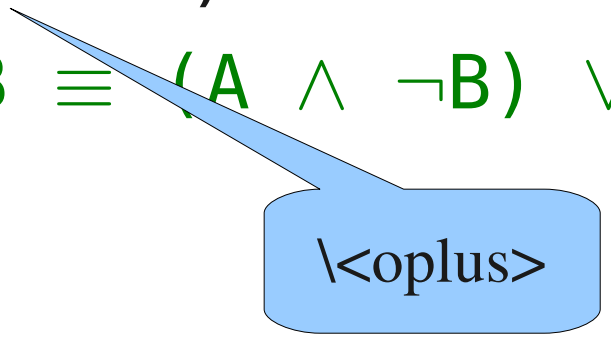

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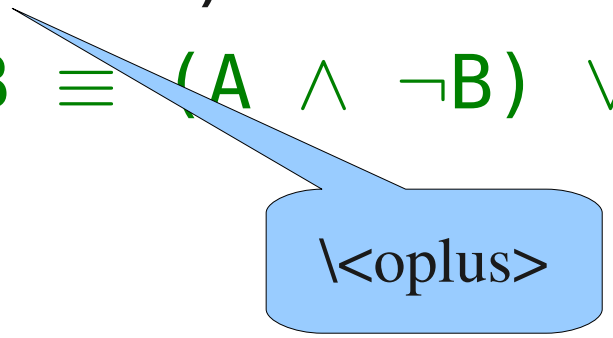

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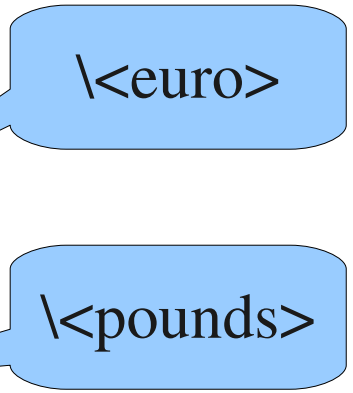

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\<euro>

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Now `€ 10` means `Euro 10`.

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Abbreviations do not replace definitions!

Document Preparation

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Proper typesetting of mathematical symbols

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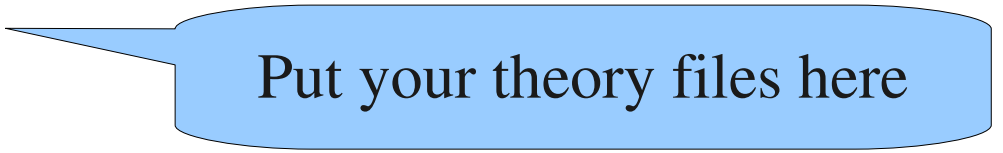

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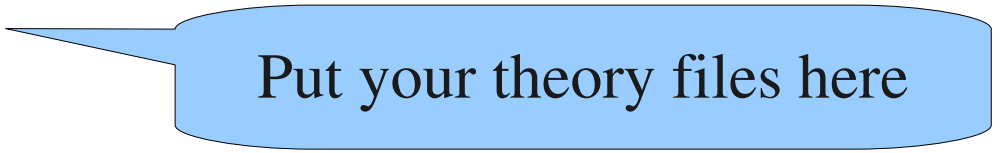

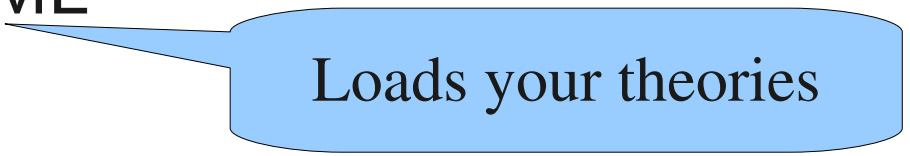

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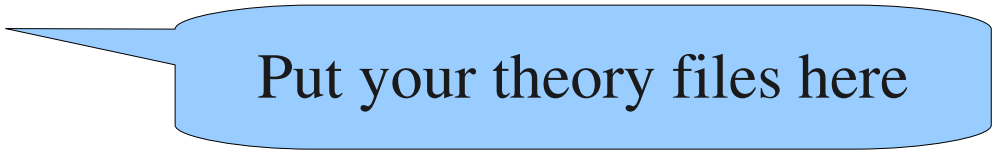

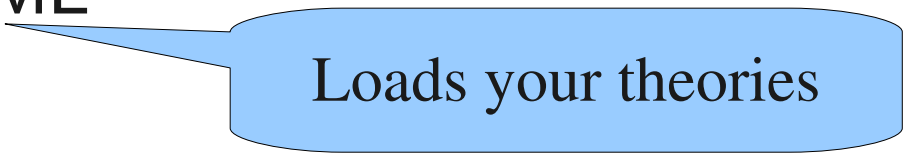

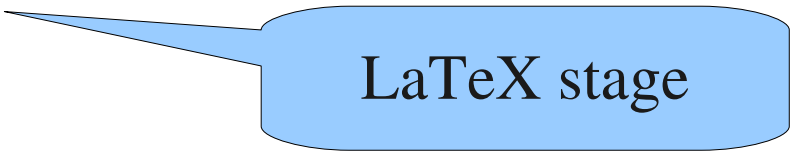

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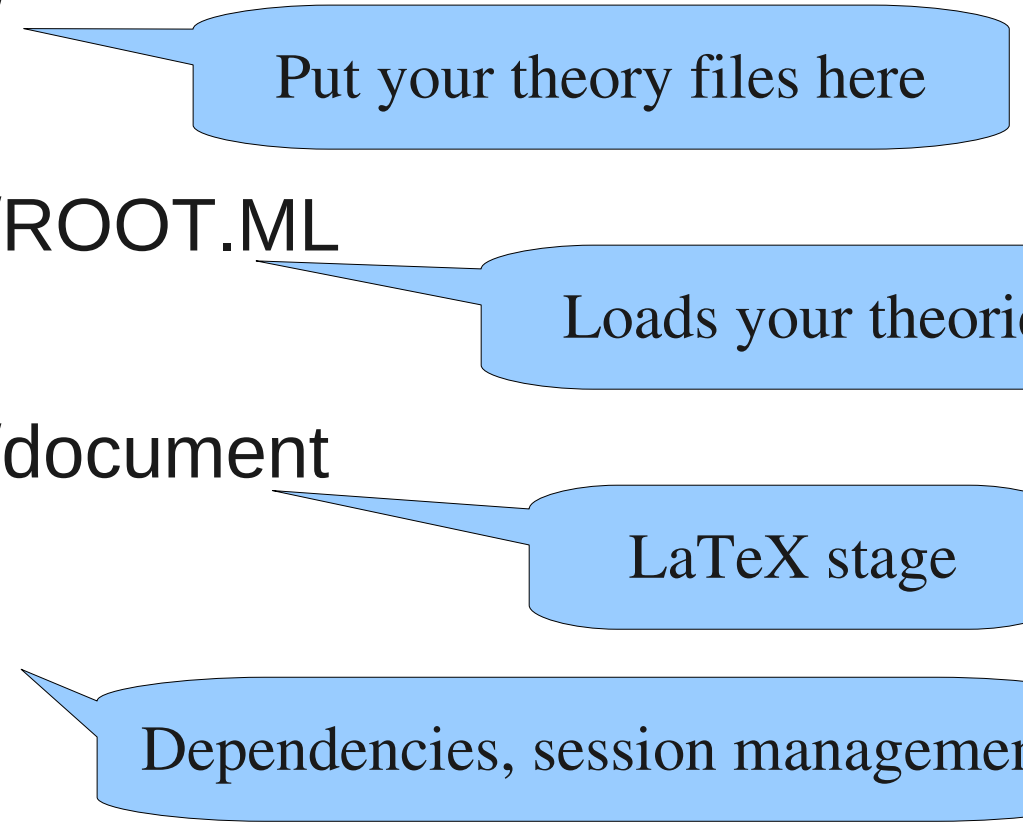

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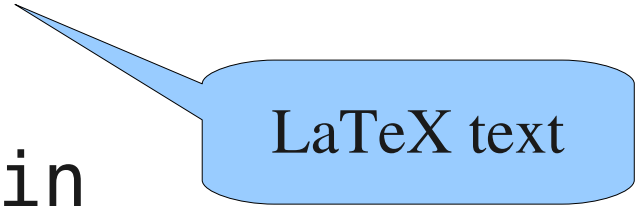

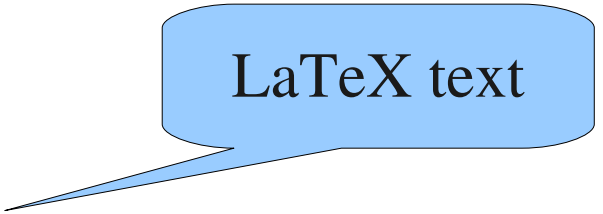

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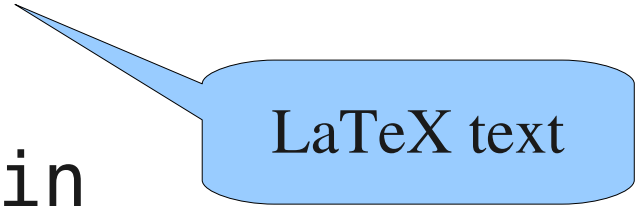

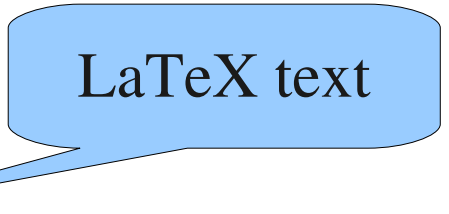

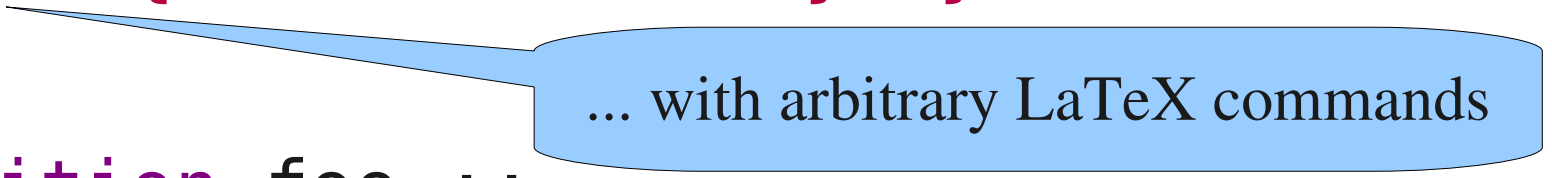

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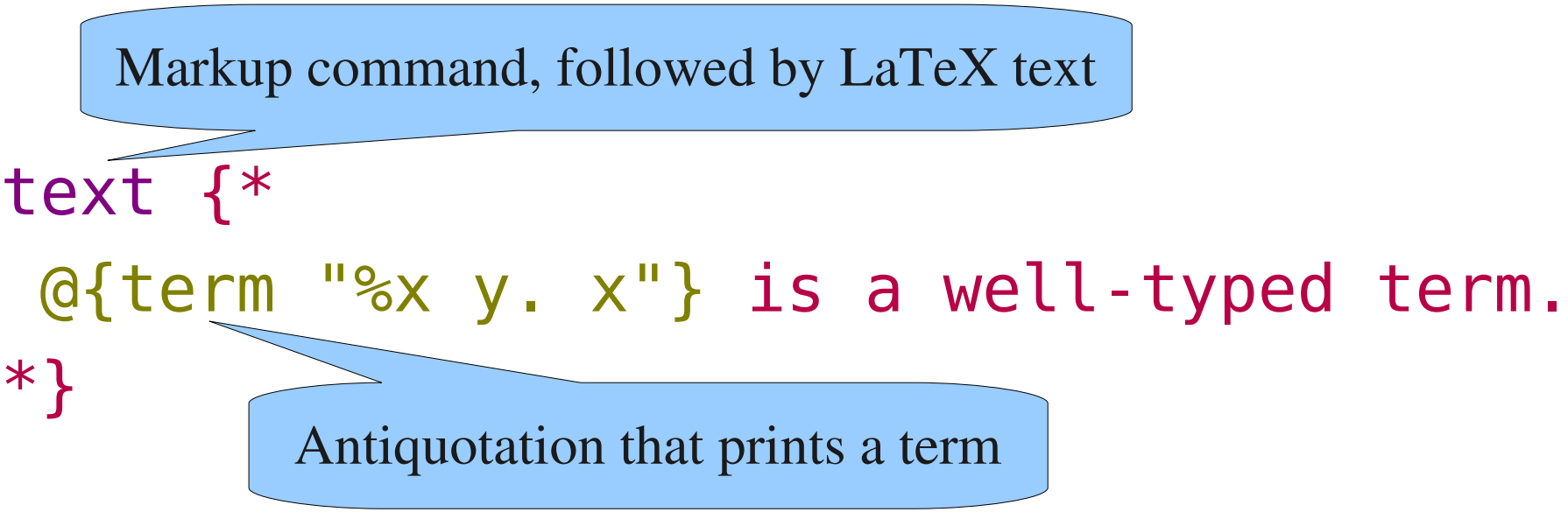

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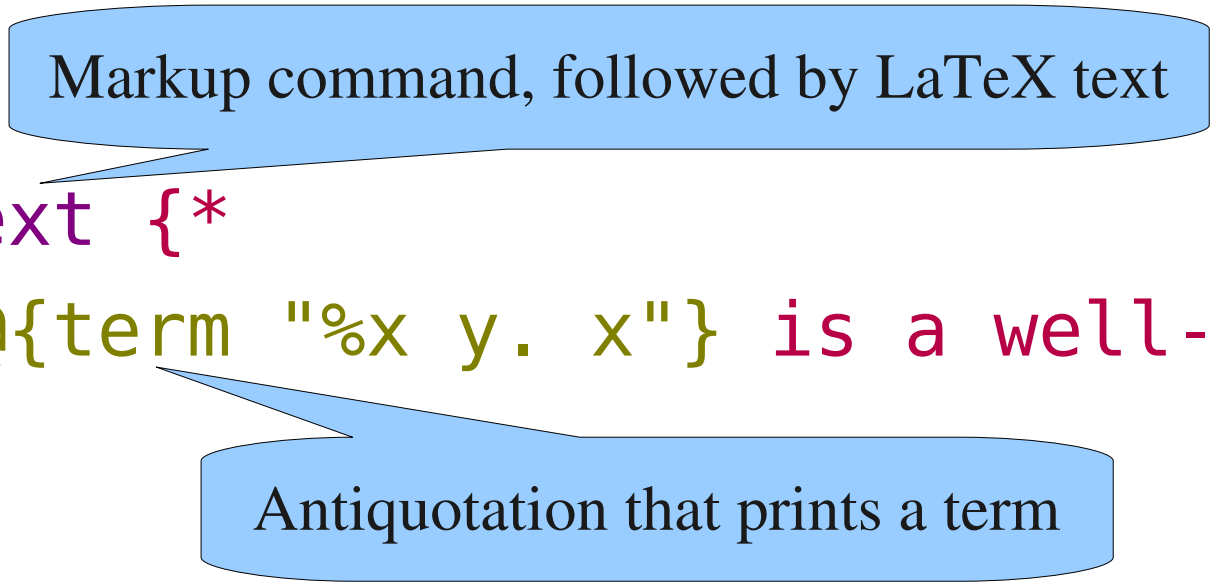

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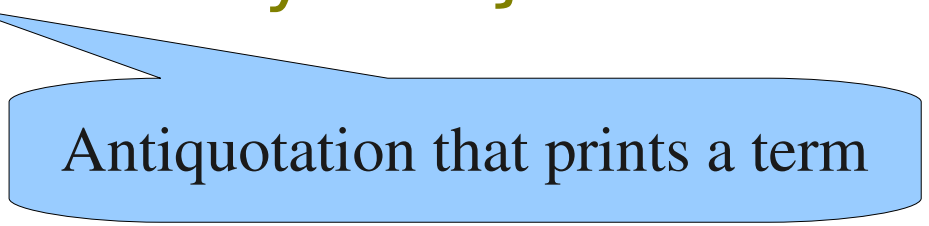

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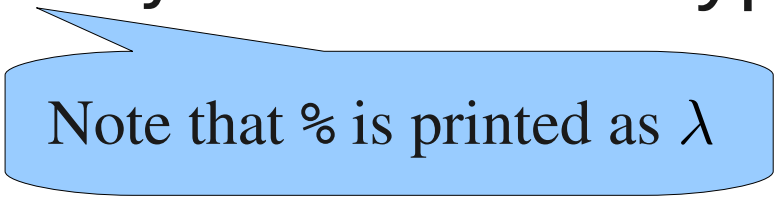

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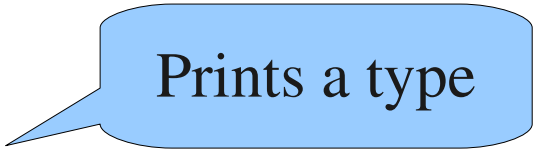

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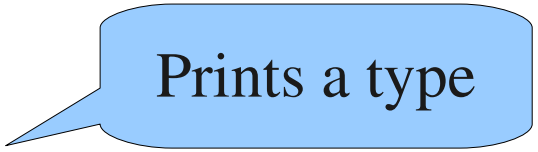

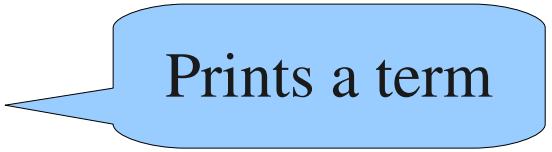

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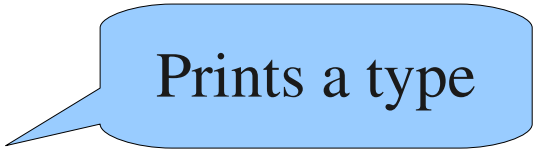
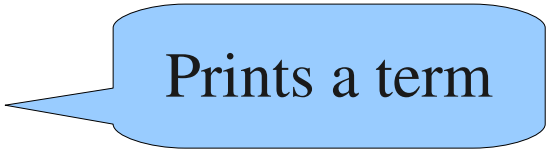
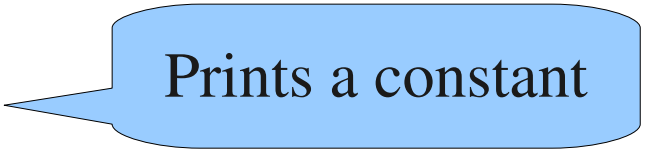
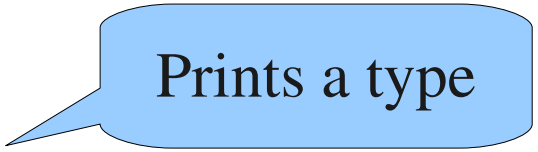


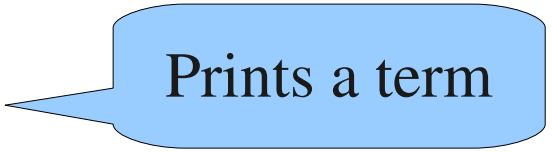


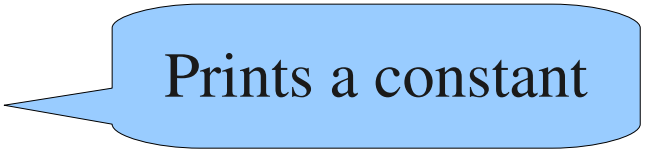


Useful Antiquotations

- `@{typ "τ"}`
- `@{term "t"}`
- `@{const "c"}`
- `@{prop "ϕ"}`
- `@{thm name}`
- `@{text "s"}`







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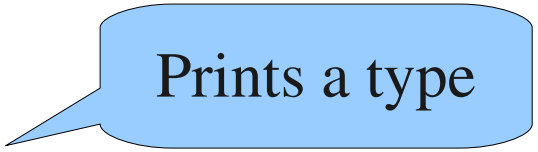
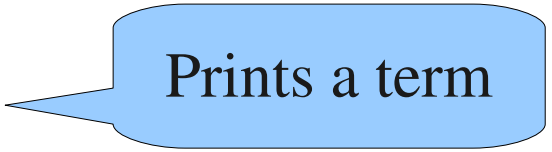
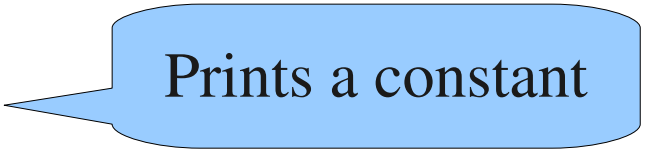






Useful Antiquotations

- `@{typ "τ"}` — Prints a type
- `@{term "t"}` — Prints a term
- `@{const "c"}` — Prints a constant
- `@{prop "ϕ"}` — Prints a proposition
- `@{thm name}` — Prints a theorem
- `@{text "s"}` — Prints uninterpreted text

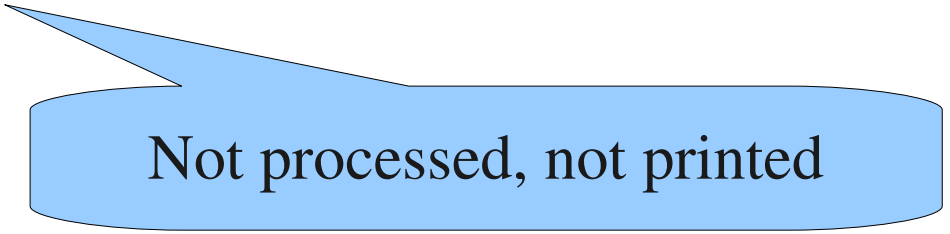
Suppressing Output

- `(* This is a comment. *)`

- `theory Foo (*<*)imports Main(*>*) begin`

Suppressing Output

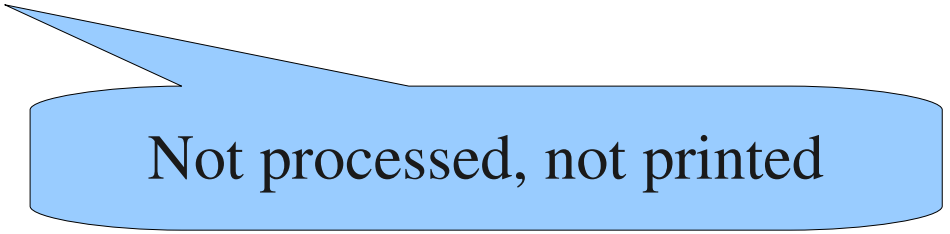
- `(* This is a comment. *)`



- `theory Foo (*<*)imports Main(*>*) begin`

Suppressing Output

- `(* This is a comment. *)`



- `theory Foo (*<*)imports Main(*>*) begin`